WHITE PAPER

THE PHYSICS OF A COLLISION

When we drive our vehicles down the road, we are driving them at a certain velocity and direction. All vehicles have weight and mass to the order of a few hundred pounds for a motorcycle, a few thousand pounds for a compact car, an SUV, luxury car or several tons for a tractor-trailer truck. When our vehicles move at speed down the roadway, they create energy by virtue of their mass and forward velocity. This energy is called Kinetic Energy mathematically described as $KE = 1/2mv^2$ where m = mass (lb_m) and v = velocity (ft/s). Assuming the mass of a vehicle is assumed to be constant, it will create a certain quantity of kinetic energy while in motion at speed. When one accelerates a vehicle to a higher velocity, the vehicle's kinetic energy increases by the square of the velocity divided by 2. For example, if we consider a vehicle that weighs approximately 4000 pounds that is traveling at 60 mph (88 ft/s), this vehicle will produce approximately 480,994 $ft•lb_f$ of kinetic energy. If the velocity of this vehicle is increased to 70 mph (102.67ft/s) which is a 10 mph increase and 16.7% increase in speed, the vehicle's kinetic energy increases to 654,728 $ft•lb_f$ which is an increase of approximately 36.1%.

The physics of a collision between two bodies is governed by Newton's Second Law of Motion $F = ma/g_c$ where F = force in pounds force or (lb_f), m = mass in pound mass (lb_m), a = acceleration in feet per second squared or (ft/s^2) and g_c = a proportionality constant of $32.2(lb_m•ft/lb_f•s^2)$. When vehicles are moving at interstate highway speeds, they possess much kinetic energy. When they collide, the time of impact is very short. An example is when we watch a NASCAR race on TV, and a race car collides with a retaining wall or another race car at say 150 mph. The forces generated due to these very short impact time intervals are called impulsive forces. Newton's Second Law of motion can be restated algebraically as $F = ma/g_c = mv/tg_c$ where v = velocity in ft/s, t = time in seconds or s and v/t = “a” the average acceleration in ft/s^2. Due to this very brief elapsed time interval during a collision, it can be seen mathematically from this equation that this very brief impact elapsed time interval causes these instantaneous impact forces between two vehicle bodies to be very high. This is due to the time variable "t" being in the denominator of the equation. Forces can be generated at impact to the order of tens of thousands to hundreds of thousands of pounds depending on the mass of the vehicles and their velocity at impact.

For example, if we have two identical vehicles that weigh 4000 pounds each and they collide in an opposite direction also known as a head on collision in a direct central impact where each vehicle is moving at say 55 mph, the combined closing velocity of the two vehicles at impact is 110 mph or 161.33 ft/s. When they collide with each other head on, both vehicle bodies absorb kinetic energy, collapse and crush inward. Let’s assume for this discussion that the frontal crush measurement of each vehicle measures to be approximately three feet where the total crush of both vehicles is six feet. From the time the two vehicles first touch each other until maximum body compression and collapse is achieved, each vehicle slows very quickly from 55mph to 0mph. The average speed of each vehicles during the collision is (80.67ft/s + 0ft/s)/2 = 40.33ft/s. The average elapsed time of impact for one vehicle can be calculated using the distance formula {d = vt} where we algebraically solve this equation for the time variable "t" or t = d/v. We then substitute the numbers in and get (3ft)/(40.33 ft/s) = 0.074s or 74 milliseconds. Using these assumptions and average collision time calculation, we can then calculate the average force generated during the collision on each vehicle to be approximately:

$$F = mv/tg_c = (4000lbm)(40.33 ft/s)/(((32.2(lbm•ft/lbf•s))(0.074s)) = 67,702 \text{ pounds.}$$

This would be 67,702 lbs acting on each vehicle over a period of 74 milliseconds. Based on this calculation, we can see why a motor vehicle's body structure experiences the damage it incurs during a highway speed collision.

Newton's Law above can be rearranged into the form of $\int f \cdot dt = \int m(dv/dt)dt$ which is called the impulse of the force. We can then apply integral calculus and integrate both sides of this equation to obtain $f(t_2 - t_1) = mv_2 - mv_1$ for a single mass. The term $(t_2 - t_1)$ in this resulting equation is the elapsed time of impact for a mass, and the term mv is called the linear momentum of the mass where Isaac Newton called this term the quantity of motion. The term $(mv_2 - mv_1)$ equals the change in linear momentum of this mass during a collision.

When two vehicles collide with each other, the force that Vehicle A generates against Vehicle B is equal in absolute value to the force that Vehicle B generates against Vehicle A during the impact. These forces are equal in magnitude but are opposite in direction. There is a theoretical and instantaneous point during the impact between the two vehicles where there is no relative motion between them which is the point of maximum force and body compression. Assuming this, we can develop another equation utilizing the impulse equation $\int f \cdot dt$ for each vehicle and set them equal to each other where one side of the equation must have a negative sign to indicate that the impulse of Vehicle A is opposite in direction to Vehicle B or $\int f_A \cdot dt = -\int f_B \cdot dt$. Substituting we have $(m_A \cdot v_{A2} - m_A \cdot v_{A1}) = (m_B \cdot v_{B1} - m_B \cdot v_{B2})$. This equation can then be rearranged algebraically to obtain $(m_A \cdot v_{A1} + m_B \cdot v_{B1}) = (m_A \cdot v_{A2} + m_B \cdot v_{B2})$. This new equation is called the Conservation Law of Linear Momentum and is used to calculate the before impact velocities of two vehicles that collide with one another. Linear Momentum is always conserved in a collision because atomic, gravitational forces, frictional forces from skidding tire and air drag on the vehicle bodies are very small in comparison to the very high contact forces generated during a collision. So these very small secondary forces can be assumed insignificant. Momentum is always conserved, and the vector sum of the two impulses is zero. The Conservation Law of Linear Momentum equation states that the combined momentum of both vehicles before a collision is equal in magnitude to the combined momentum of both vehicles after the collision. Since traffic accidents are usually bidirectional in nature (the more severe vehicle collisions), this equation must be utilized in both the x and y directions for the two vehicles by incorporating a branch of mathematics called trigonometry which is the study of vector algebra. A vector by definition is defined as a quantity that has both magnitude and direction. Because of this bidirectional nature of a collision between two vehicles, two equations one for the x direction and one for the y direction are created that generate a system of two equations with four unknown variables which are the before and after impact velocities of each vehicle. The masses however will be known from vehicle data. Once the direction vectors of both vehicles are measured and determined before and after the collision takes place at the accident scene as well as the after impact velocities of both vehicles are calculated as described below, then these two x and y Conservation Law of Linear Momentum equations can then be solved simultaneously to obtain the before impact velocities of the two vehicles.

There are some limitations however in using the Conservation of Linear Momentum equation. It can only be used to calculate the velocities of two vehicles that have similar masses. If the mass of one vehicle is huge compared to the other vehicle say for example a fully loaded 80,000 pound tractor trailer truck compared to a small Honda Fit that weighs around 2800lbs, the mass ratio would be 28.5 to1. Another limitation would be if the vehicles collide with each other at a very acute angle say 10° to 15°. The Conservation of Linear Momentum equation will then become very sensitive and can give velocities before impact that are very high which is not realistic.

An energy balance must be calculated for each vehicle in order to obtain the after impact velocity of the vehicles involved in a collision. This energy balance must capture the kinetic energy of the vehicle, the work energy of the vehicle’s skid and rotational energy from a yaw, pitch or roll. This energy balance in equation form is: $1/2mv^2_2 = 1/2mv^2_1 + N \cdot \mu \cdot d + 1/2I\omega^2$. It states that a vehicle’s kinetic energy just as it separates from the other vehicle during a collision ($1/2mv^2_2$) is equal to some intermediate velocity of that vehicle ($1/2mv^2_1$) plus the work energy of the vehicle’s tires or body skidding across one or more surfaces ($N \cdot \mu \cdot d$) plus the vehicle’s rotational energy created during a yaw, pitch or roll ($1/2I\omega^2$). When a vehicle's tires and/or body panel slide over various surfaces, the vehicle's kinetic energy is transformed into abrasive damage of the tires and body panels and also heat energy by way of frictional heating. This type of energy conversion is not conserved because the heat energy is released into the atmosphere and is lost, and the damage to the vehicle is non-reversible. The frictional forces acting on the vehicle to slow and stop it are the primary forces involved. Air drag is considered to be insignificant and is not included in the calculations because the vehicles slow very quickly to their rest positions (within about 2 seconds) after a collision. The term $1/2I\omega^2$ can be used in the equation if a vehicle yaws, pitches or rolls about its center of mass after a collision and accounts for rotational energy where ω = rotational velocity in radians per second (rad/s) about a vehicle's x,y or z axis and I = the vehicle mass moment of inertia (lb_f - s^2 - ft) about a vehicle's x,y or z axis. When a vehicle skids across various surfaces, work is done to the vehicle to slow it to a stop. The equation for the frictional forces developed by a skidding tire or body panel on a surface is equal to N•μ•d. N = the Normal Force or perpendicular force of vehicle's weight over the tire or body panel; μ = the dynamic coefficient of friction or (drag factor) between the tires or body panel and the surface it is skidding on; d = the distance in feet the vehicle slides after impact until it reaches its final rest position and becomes stationary. N is dependent on the angle or slope of the road or ground surface. For example, if the roadway surface has a significant slope say a 30% grade, then the angle of the road surface would be approximately 16.7°. This affects the amount of perpendicular vehicle weight acting over the tires or body panel. If a 1000 pounds of vehicle weight acts over a wheel on a level surface, then the perpendicular component of the vehicle's weight over the wheel at an inclination of 16.7° is reduced to 958 pounds which is a 4.2% reduction in load over the wheel. This can be calculated by performing a vector analysis using trigonometry.

A vehicle's Drag Factor must also be calculated in traffic accident reconstruction. This term is not mentioned in engineering mechanics or engineering physics textbooks. Drag Factor (f) and Coefficient of Friction (μ) are not the same. Drag Factor "f" is the deceleration coefficient for a vehicle. The Coefficient of Friction "μ" is the deceleration coefficient for a sliding tire. The Drag Factor "f" and Coefficient of Friction "μ" are the same, if and only if, a motor vehicle is sliding on a level surface with all four wheels locked and skidding. The equation for a vehicle's Drag Factor is defined as f = a/g. However the equation for a vehicle's individual tire is $\mu = F_f/w$ where "F_f" = drag force in pounds (lbs) to slow or retard the tire's motion and "w" = vehicle's weight over the tire in pounds (lbs). Regarding f = a/g, "a" is equal to the deceleration rate of the vehicle in units of ft/s², and "g" is the acceleration due to the earth's gravity in units of ft/s². Both a vehicle's drag factor and a tire's coefficient of friction are unitless numbers since the units algebraically divide out in these equations. If a vehicle decelerates at the rate of say 0.60g's, then the vehicle's drag factor is 0.60. Drag Factor is a calculated number that is utilized in the energy balance equation $1/2mv^2_2 = 1/2mv^2_1 + N \cdot \mu \cdot d + 1/2I\omega^2$ where "f" is substituted for "μ" and averages the frictional drag of all tires at both axles and is calculated by the engineer to represent the overall frictional retarding drag factor on the entire vehicle as it slides over various surfaces just after the collision takes place.

Northwestern University derived an equation to calculate the drag factor for an entire vehicle which is: $f_{RD} = ((f_f - x_f(f_f - f_r))/((1 - z(f_f - f_r))$. Another way to calculate the drag factor for a vehicle is to determine the weight over each wheel and also determine which wheel was locked and which wheel was not locked. Then calculate the braking/stopping force for each wheel. Sum these braking forces and divide this sum by the vehicle's curb weight plus the weight of the occupants. This will calculate the average drag factor of the vehicle. The drag factor can also be measured for a vehicle by performing skid tests utilizing a Vericom computer. The driver must setup the computer and input the necessary data before the test. It will then sense and measure the drag factor for the driver during hard braking. Another way to measure a vehicle's drag factor is to weigh the vehicle with and without the driver and any passengers. Attach a paint gun to the vehicles bumper aiming it down toward the road surface and wire it to the brake pedal switch. When the driver applies the brakes, the paint gun will shoot a spot of paint on the road surface to mark the application point of the vehicle brakes. One can also pull some heavy and brightly colored string across the roadway and nail it down. While using a stop watch, start and run the vehicle at a predetermined speed. When the vehicle's front wheels roll over the string or the paint gun marks the road surface, begin hard braking and start the stop watch until the vehicle comes to a complete stop and stop the stop watch. Then exit the vehicle and measure the stopping distance and record it. Record the elapsed time it took to bring the vehicle to a stop. Perform this about ten times. As one performs these test runs, the driver will get a feel for the vehicle and will achieve shorter and shorter stopping distances until he starts to hear the tires squalling. If the vehicle does not have ABS, the driver should make sure to modulate the brake pedal force so as not to lock the wheels but allow the tires to roll and squall somewhat at the same time. This will be when maximum braking is achieved. A test can also be performed to lock the wheels during the skid test and determine the drag factor of a sliding vehicle. You can then use the measured data and calculate the vehicle's average drag factor by using the equation: $f = V^2/(2gd)$. The deceleration rate can then be calculated by using $a = fg$. The measured elapsed time can be used to calculate the deceleration rate as a check. The most accurate way to determine when the driver begins hard braking is to instrument to vehicle with a bumper mounted paint gun that shoots a spot of paint on the roadway when the brakes are pressed by the driver. From the point where the paint is applied to the roadway at the test site, actual braking begins forward of the paint spot a few feet which must be determined.

The resultant drag factor calculation is very important when a vehicle skids out of control after a collision. As a vehicle skids after impact, the drag forces generated by each tire can vary. When a vehicle slides from one surface to another, for example from the asphalt to a grass/dirt shoulder, the drag forces will vary for each tire as the individual tire slides from the asphalt to the dirt shoulder. The vehicle's tire drag forces are maximum during a forward skid over a paved surface with all wheels locked. It has been determine by the tire manufacturers that tires will generate a slightly higher COF when in a lateral (sideways skid). A vehicle's tire drag is minimum (due to rolling friction only) when the vehicle is free rolling forward or backward during its movement in a yaw (assuming the wheels are not locked due to impact damage). If one or more wheels of an axle are locked after a collision due to damage, then the locked wheel will create more drag on an axle than the free rolling wheel and this condition must be taken into consideration in calculating the vehicle's drag factor. So the vehicle's frictional drag varies as the vehicle yaws and translates about its center of mass (assuming the vehicle goes into a yaw spin after a collision). According to Northwestern University, if the vehicle is skidding on an inclined surface, then the drag factor on a grade is $f_{RD} = (\mu + G)/\sqrt{}SRT(1 - G^2)$ where "G" is the grade of the surface expressed as a decimal and carries a positive sign for an incline surface and a negative sign for a decline surface. The above drag factor equations were stated with permission from the Northwestern University Traffic Accident Reconstruction Manual Volume 2 1990.

In some cases, the collision forces are so violent that a vehicle's body can be literally torn into two large pieces. In these cases, the direction vector and movement distances of each vehicle body half must be measured and determined. The after impact velocity must be calculated for each vehicle body half. The engineer will then sum the two velocity vectors for each vehicle body half which allows the engineer to calculate the velocity vector of the center of mass of the vehicle which is equal to the velocity of the vehicle when it has been torn into two pieces.

Once the after impact velocities of both vehicles have been determined as stated in the above paragraphs, these velocity numbers along with the vehicle masses and their vector angles must then be plugged into the x and y Conservation Law of Linear Momentum equations. This generates two equations; one for the x direction and one for the y direction where the before impact velocities of both vehicles become the two unknown variables. These two equations then become a system of two linear equations with constant coefficients and two unknowns. They can then be solved simultaneously to obtain the before impact vehicular velocities vectors for each vehicles. This calculation must then be checked algebraically to make sure that each side of both equations matches numerically. If the check shows that each side of the equation does not match within say 1.0 (due to the use of decimal approximations of the after impact vehicle velocity vectors and their angles in the equations), then an error has been made during the reconstruction efforts, and the collected data must be reviewed to resolve the issue.

Another equation can be developed for use in traffic accident reconstruction. This equation is derived from the impulse definition $\int f \cdot dt$ and the assumption of a direct central impact which means that both masses strike each other with their centers of mass in line with each other. During the derivation, the collision is made up of two subintervals of time, the period of deformation and the period of restitution. The period of deformation refers to the time duration of a collision that starts from the initial contact of the two vehicles and ends at the instant of maximum body deformation or compression. It is assumed at this instant of maximum body deformation and compression that there is no relative momentary movement between the two vehicles at their contact point. During this subinterval of time, the impulse is defined as $\int D \cdot dt$. The subinterval of restitution is the time interval from maximum body deformation and compression to the instant where the two masses just separate and is defined as $\int R \cdot dt$. Both of these impulses are equal in absolute magnitude but are opposite in direction during the point of maximum body compression. If two bodies are perfectly elastic, they will bounce off of each other and reestablish their original shape during the subinterval of restitution like a pair of billiard balls. However, when two bodies do not reestablish their shapes, we say that plastic deformation has occurred. The ratio of the impulse during restitution to the impulse of deformation $\int R \cdot dt / \int D \cdot dt$ is equal to a number called the Coefficient of Restitution (COR) and is denoted by the greek letter ε. It can be shown algebraically that this equation equals $\varepsilon = -(V_{B2} - V_{A2})/(V_{B1} - V_{A1})$ and defines the negative relative velocity of separation to the relative velocity of approach. The COR range for an impact between two bodies is between 0 and 1. For an ideal and perfectly elastic collision where both bodies collide, rebound and regain their initial shape before the impact took place like the example of the two billiard balls, the COR = 1. For a totally plastic collision where two bodies would collide and stick together like soft putty, the COR = 0. The COR for two modern passenger vehicles that collide with each other depend on the speed and orientation at which they impact each other. According to Randall Noon in his 1994 book "Engineering Analysis of Vehicular Accidents", laboratory research has shown through fixed barrier impact studies that the COR can range from 0.20 with a speed of 25 mph, 0.1 with a speed of 35 mph and 0.004 with speeds of over 50 mph. Unless impacts studies of an actual collision are performed in a laboratory, which is not practical due to the cost involved in destructive testing, then the COR will not be known exactly and can only be estimated and based on prior laboratory tests and texts.

Motor vehicle crash reconstruction is a complex and very involved undertaking. This paper was written to discuss only the basic engineering physics of impact between two vehicles to determine impact velocities of two similarly sized motor vehicles. The attached pages are the “Foreword, Preface and “Tables of Contents” from the textbooks of Northwestern University Center For Public Safety and will give the reader the scope concerning a full traffic crash reconstruction. Northwestern University was the first educational institution in the United States to organize and develop the science of motor vehicle crash reconstruction.

---End of Paper---